BRS Cohomology of Zero Curvature Systems
I. The Complete Ladder Case

M. Carvalho, L.C.Q. Vilar, C.A.G. Sasaki

C.B.P.F
Centro Brasileiro de Pesquisas Físicas,
Rua Xavier Sigaud 150, 22290-180 Urca
Rio de Janeiro, Brazil

and

S.P. Sorella

UERJ
Departamento de Física Teórica
Instituto de Física, UERJ
Rua São Francisco Xavier, 528
20550-013, Rio de Janeiro, Brazil
and
C.B.P.F
Centro Brasileiro de Pesquisas Físicas,
Rua Xavier Sigaud 150, 22290-180 Urca
Rio de Janeiro, Brazil

REF. CBPF-NF-062/95

Abstract

We present here the zero curvature formulation for a wide class of field theory models. This formalism, which relies on the existence of an operator δ which decomposes the exterior space-time derivative as a BRS commutator, turns out to be particularly useful in order to solve the Wess-Zumino consistency condition. The examples of the topological theories and of the B-C string ghost system are considered in detail.

1 Introduction

Nowadays it is an established fact that the search of the possible anomalies and of the counterterms which arise at the quantum level in local field theories can be done in a purely algebraic way[1] by identifying the cohomology classes of the nilpotent BRS operator b in the space of the integrated local polynomials in the fields and their derivatives. This means that one has to look at the nontrivial solutions of the equation

$$b \int \omega_D^G = 0 \ , \tag{1.1}$$

ω_D^G denoting a local polynomial in the fields of ghost number G and form degree D, D being the dimension of the space-time. The cases $G = 0, 1$ correspond respectively to counterterms and anomalies.

The BRS consistency condition (1.1), when translated at the nonintegrated level, yields a system of equations usually called descent equations (see [1] and refs. therein)

$$\begin{aligned}
&b\,\omega_D^G \quad + d\,\omega_{D-1}^{G+1} = 0 \ , \\
&b\,\omega_{D-1}^{G+1} + d\,\omega_{D-2}^{G+2} = 0 \ , \\
&\qquad \ldots\ldots\ldots\ldots \\
&\qquad \ldots\ldots\ldots\ldots \\
&b\,\omega_1^{G+D-1} + d\,\omega_0^{G+D} = 0 \ , \\
&b\,\omega_0^{G+D} = 0 \ ,
\end{aligned} \tag{1.2}$$

$d = dx^\mu \partial_\mu$ being the exterior space-time derivative and $\omega_j^{G+D-j}(\, 0 \le j \le D)$ local polynomials of ghost number $(G + D - j)$ and form degree j. The operators b and d obey the algebraic relations

$$b^2 \; = \; d^2 \; = \; b\,d + d\,b \; = 0 \ . \tag{1.3}$$

It should be remarked that at the nonintegrated level one looses the property of making integration by parts. This implies that the fields and their derivatives have to be considered as independent variables.

The problem of solving the descent equations (1.2) is a problem of cohomology of b modulo d, the corresponding cohomology classes being given by solutions of (1.2) which are not of the type

$$\begin{aligned}
&\omega_m^{G+D-m} = b\,\hat{\omega}_m^{G+D-m-1} + d\,\hat{\omega}_{m-1}^{G+D-m} \ , \qquad 1 \le m \le D \\
&\omega_0^{G+D} = b\,\hat{\omega}_0^{G+D-1} \ ,
\end{aligned} \tag{1.4}$$

with $\hat{\omega}$'s local polynomials.

[1] For a recent account on the so called *Algebraic Renormalization* see [1].

Recently a new method for finding nontrivial solutions of the tower (1.2) has been proposed by one of the authors [2] and successfully applied to a large number of field models such as Yang-Mills theories [3], gravity [4][2] , topological field theories [7, 8, 9], string [10] and superstring [11] theories, as well as W_3-algebras [12]. The method relies on the introduction of an operator δ which allows to decompose the exterior derivative as a BRS commutator, *i.e.*

$$d = -\,[b, \delta] \ . \tag{1.5}$$

It is easily proven that, once the decomposition (1.5) has been found, repeated applications of the operator δ on the polynomial ω_0^{G+D} which solves the last of the equations (1.2) will give an explicit nontrivial solution for the higher cocycles ω_j^{G+D-j}.

One has to note that solving the last equation of the tower (1.2) is a problem of local cohomology instead of a modulo-d one. Moreover, the former can be systematically attacked by using several methods as, for instance, the spectral sequences technique [13]. It is also worthy to mention that the solutions of the descent equations (1.2) obtained via the decomposition (1.5) have been proven to be equivalent to those provided by the transgression procedure based on the so called *Russian Formula* [14, 15].

The aim of the present work is twofold. First, to improve and extend the results obtained in [2] and, secondly, to discuss the deep relation between the existence of the operator δ entering the decomposition (1.5) and the possibility of encoding all the relevant informations (BRS transformations of the fields, BRS cohomology classes, solutions of the descent equations) into a unique equation which takes the form of a generalized zero curvature condition:

$$\widetilde{\mathcal{F}} = \widetilde{d}\widetilde{\mathcal{A}} - i\widetilde{\mathcal{A}}^2 = 0 \ . \tag{1.6}$$

The operator $\widetilde{d}$ and the generalized gauge connection $\widetilde{\mathcal{A}}$ in eq.(1.6) turn out to be respectively the δ-transform of the BRS operator b and of the ghost field c corresponding to the Maurer-Cartan form of the underlying gauge algebra

$$\begin{aligned} &\widetilde{d} = e^{\delta}\, b\, e^{-\delta} = b + d + ... \ , \qquad \widetilde{d}^2 = 0 \ , \\ &\widetilde{\mathcal{A}} = e^{\delta}\, c = c + ... \ . \end{aligned} \tag{1.7}$$

The main purpose of this work will be that of clarifying the meaning of the generalized gauge connection $\widetilde{\mathcal{A}}$ and of the dots .... appearing in equation (1.7) with the help of several examples.

In particular, as we shall see, the zero-curvature condition (1.6) immediately yields the cohomology classes of the generalized nilpotent operator[3] $\widetilde{d}$ [16]. The

[2]In the case of gravitational theories, the decomposition $d = -\,[b, \delta]$ was in fact already observed, see for instance refs. [5, 6].

[3]The nilpotency of $\widetilde{d}$ is a direct consequence of the zero curvature condition (1.6).

latters turn out to be naturally related to the solutions of the descent equations (1.2). In other words, once the zero curvature condition of the model under consideration has been established, the problem of finding the anomalies and the invariant actions becomes straightforward to be solved.

For the sake of completeness and in order to present several detailed models, the paper has been splitted in two parts, referred as part I and part II. This division corresponds to two different situations, called respectively the *complete* and the *noncomplete* ladder case. In the first case the components of the generalized gauge connection $\widetilde{\mathcal{A}}$ form a ladder of fields which span all possible form degrees compatible with the space-time dimension D. This means that, ordering the components of $\widetilde{\mathcal{A}}$ according to their increasing form degree p, the allowed interval, *i.e.* $0 \leq p \leq D$, is fully covered. Instead, in the noncomplete ladder case the maximum form degree reached by the components of $\widetilde{\mathcal{A}}$ is strictly lower than the space-time dimension D, *i.e.* $0 \leq p < D$.

Examples of models belonging to the first case are, for instance, the topological models of the Schwartz type such as the Chern-Simons and the BF models [17], and the B-C ghost system of the bosonic string theory [18]. On the other hand, the Yang-Mills type theories can be accommodated in the noncomplete ladder case.

Even if many properties of the models covered by the complete ladder case have already been investigated [8, 19], their zero curvature formulation still represents a very elegant and interesting aspect. Moreover, in the noncomplete case, the zero curvature condition (1.6) requires the existence of a set of new operators $(\mathcal{G}^{1-k}_k,\ 2 \leq k \leq D)$ which are in involution, i.e. the operator $\mathcal{G}^{1-k}_k$ is generated by the commutator between $\mathcal{G}^{2-k}_{k-1}$ and the operator δ of (1.5), according to the recursive formula

$$
\begin{aligned}
\mathcal{G}^{-1}_2 &= \frac{1}{2}[\,\delta\,,\,d\,]\ , \\
\mathcal{G}^{1-k}_k &= \frac{1}{k}\left[\,\delta\,,\,\mathcal{G}^{2-k}_{k-1}\right]\ , \qquad k > 2\ .
\end{aligned}
\tag{1.8}
$$

This structure naturally reminds us to the recursive construction of the Lax pair operators of the integrable systems [20]. This is a quite welcome and attractive feature which may signal a deeper relation between the BRS cohomology techniques and the integrability. Needless to say, the zero curvature condition represents in fact one of the most important chapters of the integrable systems (see also the recent works of ref. [21]).

The part I of the paper ialgebraic set up is presented. In Sect. 3 we discuss the geometrical meaning of the zero curvature condition. Sections 4 and 5 are devoted respectively to the computation of the BRS cohomology and to characterize the solution of the descent equations. Sect. 6 deals with the coupling with matter fields in the context of the BF models. Without entering in details, let us briefly comment that, in analogy with the gauge ladder $\widetilde{\mathcal{A}}$, the matter fields can be introduced

by means of a second complete ladder $\widetilde{\mathcal{B}}$ constrained by the requirement of being covariantly constant with respect to the gauge ladder, *i.e.*

$$\widetilde{\mathcal{D}}\widetilde{\mathcal{B}} = \widetilde{d}\widetilde{\mathcal{B}} - i[\widetilde{\mathcal{A}}, \widetilde{\mathcal{B}}] = 0 \ . \tag{1.9}$$

As we shall see, condition (1.9) completely characterizes the BRS transformations of the various components of $\widetilde{\mathcal{B}}$.

Finally, Sect. 7 contains a detailed discussion of the zero curvature formulation of the B-C string ghost system.

2 The general set up

In order to present the general algebraic set up let us begin by fixing the notations. We shall work in a space-time of dimension D equipped with a set of fields generically denoted by $\{\varphi_q^p\}$, q and p being respectively the form degree and the ghost number. The components φ_q^p will be treated as commuting or anticommuting variables according to the fact that their total degree, *i.e.* the sum $(q+p)$, is even or odd. Otherwise stated, the φ_q^p's are Lie-algebra valued, $\varphi_q^p = (\varphi_q^p)^a T^a$, T^a being the hermitian generators of a compact semisimple Lie group G. Moreover, these fields are assumed to be collected into a unique generalized complete field $\widetilde{\mathcal{A}}$ of total degree one, *i.e.*

$$\widetilde{\mathcal{A}} = \sum_{j=0}^{D} \varphi_j^{1-j} = \varphi_0^1 + \varphi_1^0 + \varphi_2^{-1} + \ldots + \varphi_D^{1-D} \ . \tag{2.1}$$

The name complete, as already said in the introduction, is due to the fact that the field content of the expansion (2.1) spans all possible form degrees. In addition, eq.(2.1) shows that the generalized field $\widetilde{\mathcal{A}}$ contains a zero form with ghost number one φ_0^1, and a one-form with ghost number zero φ_1^0. These fields will be naturally identified with the Faddeev-Popov ghost field and with the gauge connection of the familiar Yang-Mills gauge transformations. Therefore $\widetilde{\mathcal{A}}$ will be called the gauge ladder and the components φ_0^1 and φ_1^0 will be denoted respectively by c and A, so that

$$\widetilde{\mathcal{A}} = c + A + \varphi_2^{-1} + \ldots + \varphi_D^{1-D} \ . \tag{2.2}$$

Finally, the functional space $\mathcal{V}$ the BRS operator b acts upon is the space of the form-valued polynomials in the fields φ_j^{1-j} and their differentials, *i.e.*

$$\mathcal{V} = \text{polynomials in } \Big(\varphi_j^{1-j},\, d\,\varphi_j^{1-j}\, ;\ \ 0 \le j \le D\Big)\,, \tag{2.3}$$

d being the exterior derivative defined[4] as

$$d\eta_p = dx^\mu \partial_\mu \eta_p \tag{2.4}$$

[4]Observe that $d\varphi_D^{1-D}$ automatically vanishes, due to the dimension of the space-time.

for any p-form

$$\eta_p = \frac{1}{p!}\,\eta_{i_1.....i_p} dx^{i_1}.....\,dx^{i_p}\ , \tag{2.5}$$

where a wedge product has to be understood.

Let us also mention that, as proven in [22, 23], the space of polynomials of forms turns out to be bigger in half to include the anomalies and the Chern-Simons type actions which, as it is well known, can be naturally written in terms of differential forms.

In order to obtain the BRS transformations of the fields belonging to the gauge ladder (2.2) we introduce the generalized operator of total degree one[5]

$$\widetilde{d} = b + d, \tag{2.6}$$

and we impose the zero curvature condition

$$\widetilde{d}\widetilde{\mathcal{A}} = i\widetilde{\mathcal{A}}^2 = \frac{i}{2}\left[\widetilde{\mathcal{A}}\,,\,\widetilde{\mathcal{A}}\right], \tag{2.7}$$

where $[a, b] = ab - (-1)^{|a||b|}ba$ denotes the graded commutator and $|a|$ is the total degree of a.

Developping equation (2.7) in components and identifying the terms with the same ghost number and form degree we obtain the following transformations

$$\begin{aligned}
&bc = ic^2\ , \\
&bA = -dc + i[c, A]\ , \\
&b\varphi_j^{1-j} = -d\varphi_{j-1}^{2-j} + \frac{i}{2}\sum_{m=0}^{j}\left[\varphi_m^{1-m}\,,\,\varphi_{j-m}^{1-j+m}\right]\ , \qquad 2 \le j \le D\ ,
\end{aligned} \tag{2.8}$$

which are easily checked to be nilpotent

$$b^2 = 0\ . \tag{2.9}$$

Notice that, as announced, the transformations of the first two components of the ladder $\widetilde{\mathcal{A}}$ are nothing but the familiar BRS transformations of the Faddeev-Popov ghost and of the Yang-Mills gauge connection.

Let us introduce now the operator δ defined by (see also refs. [1, 4, 8])

$$\widetilde{\mathcal{A}} = e^{\delta}c, \tag{2.10}$$

i.e.

$$\begin{aligned}
&\delta\,\varphi_j^{1-j} = (j+1)\varphi_{j+1}^{-j}\,, \qquad 0 \le j \le D-1\,, \\
&\delta\varphi_D^{1-D} = 0\ .
\end{aligned} \tag{2.11}$$

[5] The operators b and d rby one unit.

Its action extends on the differentials $(d\,\varphi_j^{1-j}\ ,\ 0 \le j \le D)$ as

$$\begin{aligned} &\delta\, d\,\varphi_j^{1-j} = (j+1)\, d\,\varphi_{j+1}^{-j}\ , \qquad 0 \le j \le D-2\ , \\ &\delta\, d\,\varphi_{D-1}^{2-D} = 0\ . \end{aligned} \tag{2.12}$$

It is easily verified then that, on the functional space $\mathcal{V}$, the operators b and δ obey

$$d = -\,[b,\delta]\ , \qquad [d,\delta] = 0\ , \tag{2.13}$$

i.e. δ allows to decompose the exterior derivative as a BRS commutator.

Equations (2.11), (2.12) show that the operator δ increases the form-degree by one unit and decreases the ghost number by the same amount, so that it has total degree zero. In particular from eq.(2.13) it follows that

$$\widetilde{d} = b + d = e^{\delta}\, b\, e^{-\delta}\ . \tag{2.14}$$

3 The geometrical meaning of the zero curvature condition

In the previous section the BRS transformations of the component fields φ_j^{1-j} have been obtained as a consequence of the zero curvature condition (2.7).

Conversely, it is very simple to show that, assuming the BRS transformations (2.8) to hold, the zero curvature condition can be derived as a consequence of the existence of the operator δ. Indeed, applying e^{δ} to the BRS transformation of the ghost field c, *i.e.*

$$e^{\delta} b e^{-\delta} e^{\delta} c = i\, e^{\delta}\, c^2, \tag{3.1}$$

and making use of equations (2.10) and (2.14) one gets the zero curvature condition

$$\widetilde{d}\widetilde{\mathcal{A}} = i\widetilde{\mathcal{A}}^2. \tag{3.2}$$

This is not surprising since, as it is well known, the ghost field c identifies the so called Maurer-Cartan form of the gauge group G and its BRS transformation is nothing but the corresponding Maurer-Cartan equation [24] which is in fact a zero curvature condition. This is the geometrical meaning of the equation (2.7).

4 Cohomology of the BRS operator

Even if the cohomology of the BRS operator in the case of a complete ladder field has already been studied [8], let us present here a simple derivation which may be useful for the reader.

In order to compute ththe functional space $\mathcal{V}$ we introduce the filtering operator [1, 13] $\mathcal{N}$ defined as

$$
\begin{aligned}
&\mathcal{N}\varphi_j^{1-j} = \varphi_j^{1-j} , \qquad 0 \leq j \leq D , \\
&\mathcal{N}d\varphi_j^{1-j} = d\varphi_j^{1-j} ,
\end{aligned}
\tag{4.1}
$$

according to which the BRS operator b decomposes as

$$
b = b_0 + b_1 \ , \tag{4.2}
$$

with

$$
\begin{aligned}
&b_0\, c = 0 \ , \\
&b_0\, \varphi_m^{1-m} = -d\, \varphi_{m-1}^{2-m} \ , \qquad b_0\, d\, \varphi_{m-1}^{2-m} = 0 \ , \qquad 1 \leq m \leq D ,
\end{aligned}
\tag{4.3}
$$

and

$$
b_0^2 = 0 \ . \tag{4.4}
$$

The usefulness of the above decomposition relies on a very general theorem on the BRS cohomology [13]. The latter states that the cohomology of the operator b is isomorphic to a subspace of the cohomology of b_0. We focus then on the study of the cohomology of b_0.

In particular, equation (4.3) shows that all the fields (φ_m^{1-m}, $1 \leq m \leq D$) with form degree greater than zero and their differentials are grouped in BRS-doublets [1, 13, 22]. It is known that the cohomology does not depend on such variables. Therefore the cohomology classes of b_0 depend only on the ghost field c undifferentiated, *i.e.* they are given by elements of the type

$$
\omega_{i_1.....i_n} c^{i_1} .....c^{i_n} \tag{4.5}
$$

with $\omega_{i_1.....i_n}$ arbitrary coefficients. Moreover, from the previous theorem it follows that the cohomology of b is also given by elements of the form (4.5) with, in addition, the restriction that the coefficients $\omega_{i_1.....i_n}$ are invariant tensors of the gauge group [15, 22, 23, 25].

In summary, the cohomology of the BRS operator b in the complete ladder case is spanned by invariant polynomials in the ghost field c built up with monomials of the t

$$
\mathrm{Tr} \left(\frac{c^{2n+1}}{(2n+1)!} \right) , \qquad n \geq 1 \ . \tag{4.6}
$$

5 Solution of the descent equations

Having computed the cohomology of the BRS operator b let us face now the problem of solving the descent equations

$$
\begin{aligned}
&b\,\omega_{D-j}^{G+j} + d\,\omega_{D-j-1}^{G+j+1} = 0 \ , \qquad 0 \leq j \leq D-1 \ , \\
&b\omega_0^{G+D} = 0 \ .
\end{aligned}
\tag{5.1}
$$

Introducing the generalized cocycle of total degree $(G+D)$

$$\widetilde{\omega}^{G+D} = \sum_{j=0}^{D} \omega_j^{G+D-j}, \tag{5.2}$$

the descent equations (5.1) can be cast in the more compact form

$$\widetilde{d}\,\widetilde{\omega}^{G+D} = 0 \ , \tag{5.3}$$

$\widetilde{d}$ being the nilpotent generalized differential of eq.(2.14). Taking into account the zero curvature condition

$$\widetilde{d}\widetilde{\mathcal{A}} = i\widetilde{\mathcal{A}}^2 \tag{5.4}$$

and the previous result (4.6) on the cohomology of the BRS operator b, it follows that the generalized monomials of the type

$$\mathrm{Tr}\,\frac{\widetilde{\mathcal{A}}^{2n+1}}{(2n+1)!} \ , \qquad n \geq 1 \ , \tag{5.5}$$

belongs to the cohomology of $\widetilde{d}$

$$\widetilde{d}\left(\mathrm{Tr}\,\frac{\widetilde{\mathcal{A}}^{2n+1}}{(2n+1)!}\right) = 0 \, , \qquad \mathrm{Tr}\,\frac{\widetilde{\mathcal{A}}^{2n+1}}{(2n+1)!} \neq \widetilde{d}\,\widetilde{\mathcal{Q}}^{2n} \ , \tag{5.6}$$

for any local polynomial $\widetilde{\mathcal{Q}}^{2n}$.

It is apparent thus that a solution of the descent equations (5.1) is simply provided by

$$\widetilde{\omega}^{G+D} = \mathrm{Tr}\,\frac{\widetilde{\mathcal{A}}^{G+D}}{(G+D)!} \ , \tag{5.7}$$

which, of course, is nonvanishing only if its total degree $(G+D)$ is odd. In fact, developping $(\mathrm{Tr}\,\widetilde{\mathcal{A}}^{G+D})$ according to the form-degree and to the ghost-number

$$\left(\mathrm{Tr}\,\frac{\widetilde{\mathcal{A}}^{G+D}}{(G+D)!}\right) \ = \ \sum_{j=0}^{D} \omega_j^{G+D-j} \, , \tag{5.8}$$

and recalling that $\widetilde{d}\,\widetilde{\omega}^{G+D} = 0$, it is easily verified that the ω's in eq.(5.8) obey to

$$\begin{aligned} & b\,\omega_{D-j}^{G+j} + d\,\omega_{D-j-1}^{G+j+1} = 0 \ , \\ & b\omega_0^{G+D} = 0 \ , \end{aligned} \tag{5.9}$$

i.e. they solve the descent equations.

In addition, from $(\mathrm{Tr}\,\widetilde{\mathcal{A}}^{2n+1} \neq \widetilde{d}\,\widetilde{\mathcal{Q}}^{2n})$, it follows that they provide a nontrivial solution

$$\begin{aligned} & \omega_j^{G+D-j} \neq b\,\mathcal{Q}_j^{G+D-1-j} + d\,\mathcal{Q}_{j-1}^{G+D-j} \, , \qquad 1 \leq j \leq D \, , \\ & \omega_0^{G+D} \neq b\,\mathcal{Q}_0^{G+D-1} \ . \end{aligned} \tag{5.10}$$

In particular, for the zero form ω_0^{G+D} we obtain

$$\omega_0^{G+D} = \mathrm{Tr}\,\frac{c^{G+D}}{(G+D)!}\ . \tag{5.11}$$

Let us remark, finally, that as a consequence of the fact that the generalized ladder $\widetilde{\mathcal{A}}$ is the δ-transform of the ghost field c, $\widetilde{\mathcal{A}} = e^{\delta}\, c$, the generalized cocycle (5.5) is the δ-transform of the corresponding ghost cocycle (5.11), *i.e.*

$$\left(\mathrm{Tr}\,\frac{\widetilde{\mathcal{A}}^{2n+1}}{(2n+1)!}\right) = e^{\delta}\,\mathrm{Tr}\,\left(\frac{c^{2n+1}}{(2n+1)!}\right)\ . \tag{5.12}$$

5.1 Example I: the Chern-Simons theory

For a better understanding of the previous construction let us discuss in details the case of the three dimensional Chern-Simons theory, corresponding to $G = 0$ and $D = 3$. This example will give us the possibility of clarifying the meaning of the negative ghost number components $\left(\varphi_j^{1-j}\ ,\ 2 \le j \le D\right)$ of the gauge ladder $\widetilde{\mathcal{A}}$. As we shall see, these fields turn out to be the so-called external BRS sources[6] needed in order to properly define [1] the nonlinear transformations of the gauge connection A and of tThe external sources are then naturally included in the zero curvature formalism.

In a three dimensional space-time the complete gauge ladder $\widetilde{\mathcal{A}}$ of eq.(2.2) takes the following form

$$\widetilde{\mathcal{A}} = c + A + \gamma + \tau\ , \tag{5.13}$$

γ and τ identifying respectively the negative ghost number components φ_2^{-1} and φ_3^{-2}. From the zero curvature condition (2.7) one gets the BRS transformations:

$$\begin{aligned} b\,c &= ic^2\ , \\ b\,A &= -dc + i\,[c, A]\ \ , \\ b\,\gamma &= -F + i\,[c, \gamma]\ \ , \\ b\,\tau &= -d\gamma + i\,[c, \tau] + i\,[A, \gamma]\ \ , \end{aligned} \tag{5.14}$$

F being the two-form gauge field strength $F = dA - iA^2$. As explained before, in order to find a solution of the descent equations

$$\begin{aligned} &b\,\omega_{3-j}^{j} + d\,\omega_{2-j}^{j+1} = 0\ , \qquad 0 \le j \le 2\ , \\ &b\omega_0^3 = 0\ , \end{aligned} \tag{5.15}$$

it is sufficient to expand the generalized cocycle of total degree three

$$\widetilde{\omega}^3 = \frac{1}{3!}\mathrm{Tr}\,\widetilde{\mathcal{A}}^3. \tag{5.16}$$

[6] In the framework of Batalin-Vilkovsky [26] these fields are usually called antifields.

After an easy computation we get

$$\frac{1}{3!}\mathrm{Tr}\,\widetilde{\mathcal{A}}^3 = \omega_3^0 + \omega_2^1 + \omega_1^2 + \omega_0^3 \ , \tag{5.17}$$

with

$$\begin{aligned}
\omega_0^3 &= \frac{1}{3!}\mathrm{Tr}\,c^3 \ , \\
\omega_1^2 &= \frac{1}{2}\mathrm{Tr}\,c^2 A \ , \\
\omega_2^1 &= \frac{1}{2}\mathrm{Tr}\,\left(c^2\gamma + cA^2\right) \ , \\
\omega_3^0 &= \frac{1}{2}\mathrm{Tr}\,\left(c^2\tau + cA\gamma + c\gamma A + \frac{A^3}{3}\right) \ .
\end{aligned} \tag{5.18}$$

From

$$-i\mathrm{Tr}\,\left(c^2\tau + cA\gamma + c\gamma A\right) = -\mathrm{Tr}\,AF + b\,\mathrm{Tr}\,\left(c\tau + A\gamma\right) + d\,\mathrm{Tr}\,c\gamma \ , \tag{5.19}$$

the three-form ω_3^0 can be rewritten as

$$\omega_3^0 = \frac{-i}{2}\mathrm{Tr}\,(AF + i\frac{A^3}{3}) + \frac{i}{2}b\,\mathrm{Tr}\,(c\tau + A\gamma) + \frac{i}{2}d\,\mathrm{Tr}\,c\gamma \ , \tag{5.20}$$

yielding thus the invariant action

$$S = i\int\omega_3^0 = \frac{1}{2}\int\mathrm{Tr}\,(AF + i\frac{A^3}{3}) - \frac{1}{2}b\int\mathrm{Tr}\,(c\tau + A\gamma) \ , \tag{5.21}$$

which is easily recognized to be the so-called truncated action [1] of the fully quantized Chern-Simons gauge theory. In particular, one sees that the components (γ, τ) of the gauge ladder (5.13) are the BRS external sources corresponding to the non-linear transformations of the fields A and c.

6 Coupling with matter fields

The zero curvature formalism can be extended to include the case in which the gauge fields are coupled to matter fields whose quantization requires the introduction of a complete ladder matter multiplet. A typical example of this kind of coupling is given by the topological BF systems [17, 27] whose classical action reads

$$\mathrm{Tr}\int_{\mathcal{M}^D}\mathcal{B}_{D-2}^0 F \ , \tag{6.1}$$

where F is the two-form gauge curvature, $\mathcal{B}_{D-2}^0$ is a $(D-2)$ form with ghost number zero and $\mathcal{M}^D$ a D-dimensional manifold without boundaries.

In the next section we shall discuss another example of matter system, namely the B-C ghost system of the string theory, whose action is not directly given in terms of differential forms. Nevertheless we shall see that this model, although different from the BF systems, actually shares many properties of the latters.

The inclusion of the matter fields goes as follows (see also ref. [19]): we introduce a set of fields $(\mathcal{B}_0^{D-2}, \mathcal{B}_1^{D-3}, ..., \mathcal{B}_{D-3}^1, \mathcal{B}_{D-1}^{-1}, \mathcal{B}_D^{-2})$ which together with the matter field $\mathcal{B}_{D-2}^0$ give rise to a complete ladder $\widetilde{\mathcal{B}}$ of total degree $(D-2)$, *i.e.*

$$\widetilde{\mathcal{B}} = \sum_{j=0}^{D} \mathcal{B}_j^{D-2-j} \ . \tag{6.2}$$

The BRS transformations of the various components of this ladder areobtained by requiring that $\widetilde{\mathcal{B}}$ is covariantly constant with respect to the generalized covariant derivative $\widetilde{\mathcal{D}} = \widetilde{d} - i[\widetilde{\mathcal{A}}, \ \ ]$,

$$\widetilde{\mathcal{D}}\widetilde{\mathcal{B}} = \widetilde{d}\widetilde{\mathcal{B}} - i[\widetilde{\mathcal{A}}, \widetilde{\mathcal{B}}] = 0 \ . \tag{6.3}$$

This condition, when expanded in terms of the form degree and of the ghost number, gives in fact the following nilpotent transformations:

$$\begin{aligned} b\,\mathcal{B}_0^{D-2} &= i\left[c, \mathcal{B}_0^{D-2}\right] \ , \\ b\,\mathcal{B}_j^{D-2-j} &= -d\,\mathcal{B}_{j-1}^{D-1-j} + i\sum_{m=0}^{j}\left[\varphi_m^{1-m}\,,\, \mathcal{B}_{j-m}^{D-2-j+m}\right] \ , \qquad 1 \le j \le D \ . \end{aligned} \tag{6.4}$$

Repeating the same procedure of Sect.4 and making use of the general results of refs. [15, 22, 23, 25], one easily checks that with the inclusion of the matter ladder the cohomology of the BRS operator is given by polynomials in the undifferentiated zero form ghosts $(c, \mathcal{B}_0^{D-2})$ built up with factorized monomials of the type

$$\left(\mathrm{Tr}\,\frac{c^{2n+1}}{(2n+1)!}\right)\cdot \mathrm{Tr}\,\left(\mathcal{B}_0^{D-2}\right)^m \ , \qquad m \ge 1 \ . \tag{6.5}$$

In much the same way as the gauge ladder $\widetilde{\mathcal{A}}$, the operator δ extends to the matter multiplet $\widetilde{\mathcal{B}}$ by means of

$$\widetilde{\mathcal{B}} = e^{\delta}\,\mathcal{B}_0^{D-2} \ , \tag{6.6}$$

i.e.

$$\begin{aligned} \delta\,\mathcal{B}_j^{D-2-j} &= (j+1)\mathcal{B}_{j+1}^{D-3-j} \ , \qquad 0 \le j \le D-1 \, , \\ \delta\mathcal{B}_D^{-2} &= 0 \ , \end{aligned} \tag{6.7}$$

and

$$\begin{aligned} \delta\,d\,\mathcal{B}_j^{D-2-j} &= (j+1)\,d\,\mathcal{B}_{j+1}^{D-3-j} \ , \qquad 0 \le j \le D-2 \, , \\ \delta\,d\,\mathcal{B}_{D-1}^{-1} &= 0 \ , \end{aligned} \tag{6.8}$$

so that the algebraic relations

$$d = -\,[b, \delta] \ , \qquad [\delta, d] = 0 \ , \tag{6.9}$$

are fulfilled.

For what concerns the cohomology of the generalized operator $\widetilde{d}$ of eqs.(5.4) and (6.3), it is immediately seen from eq.(6.5) that it is spanned by factorized monomials in the ladders $\widetilde{\mathcal{A}}$ and $\widetilde{\mathcal{B}}$ of the type

$$\left(\mathrm{Tr}\, \frac{\widetilde{\mathcal{A}}^{2n+1}}{(2n+1)!} \right) \cdot \left(\mathrm{Tr}\, \widetilde{\mathcal{B}}^{m} \right) \ . \tag{6.10}$$

As already discussed in the previous section, the expansion of the above expression (6.10) in terms of the form degree and of the ghost number yields a solution of the descent equations (5.1) in the presence of a matter field ladder, reproducing thus the results already established in [8]. Again

$$\left(\mathrm{Tr}\, \frac{\widetilde{\mathcal{A}}^{2n+1}}{(2n+1)!} \right) \cdot \mathrm{Tr}\, \widetilde{\mathcal{B}}^{m} = e^{\delta} \left(\mathrm{Tr}\, \frac{c^{2n+1}}{(2n+1)!} \right) \cdot \left(\mathrm{Tr}\, \left(\mathcal{B}_0^{D-2} \right)^{m} \right) \tag{6.11}$$

which shows that the cohomology of $\widetilde{d}$ is the δ-transform of that of the BRS operator b. Let us conclude this section by remarking that in a space-time of dimension $(D \geq 2)$ the gauge ladder $\widetilde{\mathcal{A}}$ contains $(D-1)$ components of negative ghost number, *i.e.* $(\varphi_2^{-1}, ..., \varphi_D^{1-D})$, while the matter ladder $\widetilde{\mathcal{B}}$ contains $(D-2)$ components with positive ghost number, *i.e.* $\left(\mathcal{B}_0^{D-2}, \mathcal{B}_1^{D-3}, ..., \mathcal{B}_{D-3}^{1} \right)$, and two components of negative ghost number, namely $\left(\mathcal{B}_{D-1}^{-1}, \mathcal{B}_D^{-2} \right)$.

These fields turn out to possess the following meaning. The set $\left(\mathcal{B}_0^{D-2}, \mathcal{B}_1^{D-3}, ..., \mathcal{B}_{D-3}^{1} \right)$ identifies the well known tower of ghosts for ghosts needed for the quantization of the BF systems. The components $\left(\varphi_3^{-2}, ..., \varphi_D^{1-D} \right)$ are then the corresponding $(D-2)$ external sources (or antifields) associated to the nonlinear transformations of the ghosts for ghosts (see eq.(6.4)), while φ_2^{-1} is the external source for the $(D-2)$ form $\mathcal{B}_{D-2}^{0}$. Finally $\left(\mathcal{B}_{D-1}^{-1}, \mathcal{B}_D^{-2} \right)$ are the sources corresponding to the first two components of the gauge ladder, i.e. c and A. We thus see that in the case of the BF systems the external sources are exchanged [8], i.e. the sources for the quantized components of the matter ladder are grouped into the gauge ladder and vice versa.

Let us also recall, for completeness, that the truncated action (including the ghosts for ghosts and the external sources) for the BF systems can be cast in the simple form [19]

$$S = \mathrm{Tr} \int_{\mathcal{M}^D} \widetilde{\mathcal{B}} \left(d\widetilde{\mathcal{A}} - i\widetilde{\mathcal{A}}^2 \right) \Big|_D^0 = - \mathrm{Tr} \int_{\mathcal{M}^D} \widetilde{\mathcal{B}}\, b\, \widetilde{\mathcal{A}} \Big|_D^0 \ , \tag{6.12}$$

where $|_D^0$ means the restriction to terms of ghost number 0 and form degree D. The equality in eq.(6.12) stems from the zero curvature condition (2.7).

In particular, using equation (6.3), expression (6.12) is easily proven to be invariant under the action of the operator b,

$$b\, S = 0 \ , \tag{6.13}$$

this equation expressing the content of the Slavnov-Taylor (or Master Equation) identity.

7 Example II: The B-C ghost system

We present here, as another interesting example of matter system, the zero curvature formulation of the two dimensional B-C model whose action reads

$$S_{B-C} = \int dz d\bar{z}\, B\, \bar{\partial} C \ , \tag{7.1}$$

where the fields $B = B_{z\,z}$ and $C = C^z$ are anticommuting and carry respectively ghost number -1 and $+1$.

It should be noted that, unlike the previous examples, the fields appearing in the action (7.1) are not naturally associated to differential forms. However we shall see that, in spite of the fact that these fields do not gsystem turns out to possess the same algebraic features of the BF models. The action (7.1) is recognized to be the ghost part of the quantized bosonic string action [7] which, as it is well know, is left invariant by the following nonlinear BRS transformations

$$\begin{aligned} s\, C &= C\, \partial\, C \ , \\ s\, B &= -\,(\partial\, B)\, C - 2\, B\, \partial\, C \ . \end{aligned} \tag{7.2}$$

In particular, the right hand-side of the BRS transformation of the field B is recognized to be the component $T_{z\,z}$ of the energy-momentum tensor corresponding to the action (7.1), this property allowing for a topological interpretation of the model.

Transformations (7.2) being nonlinear, one needs to introduce two external invariant sources $\mu = \mu^z{}_{\bar{z}}$ and $L = L_{z\,z\,\bar{z}}$ of ghost number respectively 0 and -2

$$S_{ext} = \int dz d\bar{z}\, (\mu\, s\, B + L\, s\, C) \ . \tag{7.3}$$

The complete action

$$S = S_{B-C} + S_{ext} \tag{7.4}$$

obeys thus the classical Slavnov-Taylor identity

$$\int dz d\bar{z} \left(\frac{\delta S}{\delta B} \frac{\delta S}{\delta \mu} + \frac{\delta S}{\delta L} \frac{\delta S}{\delta C} \right) = 0 = \frac{1}{2}\, b\, S \ , \tag{7.5}$$

b denoting the nilpotent linearized operator

$$b \ = \int dz d\bar{z} \left(\frac{\delta S}{\delta B} \frac{\delta}{\delta \mu} + \frac{\delta S}{\delta \mu} \frac{\delta}{\delta B} + \frac{\delta S}{\delta L} \frac{\delta}{\delta C} + \frac{\delta S}{\delta C} \frac{\delta}{\delta L} \right) \ . \tag{7.6}$$

[7] Expression (7.1) is usually accompanied by its complex conjugate. However, the inclusion of the latter in the present framework does not require any additional difficulty.

The operator b acts on the fields and on the external sources in the following way

$$
\begin{aligned}
b\,C &= s\,C = C\,\partial\,C \ , \\
b\,\mu &= \bar{\partial} C + (\partial\,\mu)\ C - \mu\ (\partial\,C)\ \ ,
\end{aligned}
\tag{7.7}
$$

and

$$
\begin{aligned}
b\,B &= s\,B = -\,(\partial\,B)\,C - 2\,B\,\partial\,C \ , \\
b\,L &= \bar{\partial}\,B - (2\,B)\,\partial\mu - \mu\partial\,B + (\partial\,L)\,C + 2\,L\,\partial\,C \ .
\end{aligned}
\tag{7.8}
$$

It should be noted that, due to the fact that the BRS transformation of B is the component $T_{z\,z}$ of the energy-momentum tensor, the differentiation with respect to the external source μ of the Legendre transformation of the complete action (7.4)

$$
Z(j,\mu,L) = S + \int dz d\bar{z}\,(j_C\,C + j_B\,B)\ \ , \tag{7.9}
$$

allows to obtain the Green functions with insertion of $T_{z\,z}$. In other words, the Slavnov-Taylor identity (7.5) is the starting point for the algebraic characterization of the energy-momentum current algebra.

Introducing now the two functional operators [10]

$$
\mathcal{W} = \int dz d\bar{z} \frac{\delta}{\delta C}\ , \qquad \overline{\mathcal{W}} = \int dz d\bar{z} \left(\mu\,\frac{\delta}{\delta C} + L\,\frac{\delta}{\delta B} \right)\ , \tag{7.10}
$$

one easily proves that

$$
\delta = dz\,\mathcal{W} + d\bar{z}\,\overline{\mathcal{W}} \tag{7.11}
$$

obeys to

$$
d = -\,[b,\delta]\ \ , \qquad [d,\delta] = 0\ , \tag{7.12}
$$

d being the exterior derivative $d = dz\partial + d\bar{z}\bar{\partial}$. We have thus realized the decomposition (2.13). In order to derive the transformations (7.7), (7.8) from a zero curvature condition we proceed as before and we define the analogue of the gauge ladder (2.2) as

$$
\tilde{C}^z = e^{\delta}\,C^z = C^z + dz + d\bar{z}\mu^z_{\bar{z}}\ . \tag{7.13}
$$

Introducing then the holomorphic generalized vector field $\tilde{C} = \tilde{C}^z\,\partial_z$, it is easily checked that equations (7.7) can be cast in the form of a zero curvature condition

$$
\tilde{d}\,\tilde{C} = \frac{1}{2}\left[\tilde{C},\tilde{C}\right] = \mathcal{L}_{\tilde{C}}\,\tilde{C}\ , \tag{7.14}
$$

where, as usual, $\tilde{d}$ is the operator

$$
\tilde{d} = e^{\delta}\,b\,e^{-\delta} = b + d\,, \tag{7.15}
$$

and $\mathcal{L}_{\tilde{C}}$ denotes the Lie derivative with respect to the vector field[8] $\tilde{C}$.

[8] Of course, the bracket $[\tilde{C},\tilde{C}]$ in eq.(7.15) refers now to the Lie bracket of vector fields.

Concerning now the second set of transformations (7.8), we define the matter ladder $\widetilde{\mathcal{B}}_{z\,z}$ as

$$\widetilde{\mathcal{B}}_{z\,z} = e^{\delta}\mathcal{B}_{z\,z} = \mathcal{B}_{z\,z} + d\bar{z}L_{z\,z\,\bar{z}} \ . \tag{7.16}$$

To expression (7.16) one can naturally associate the generalized holomorphic quadratic differential

$$\widetilde{\mathcal{B}} = \widetilde{\mathcal{B}}_{z\,z} dz \otimes dz \ . \tag{7.17}$$

Therefore, transformations (7.8) can be rewritten as

$$\widetilde{d}\,\widetilde{\mathcal{B}} - \mathcal{L}_{\widetilde{C}}\,\widetilde{\mathcal{B}} = 0 \ . \tag{7.18}$$

This equation is the analogue of the covariantly constant matter condition (6.3) and together with the equation (7.14) completely characterize the B-C system. One has to remark that, as it happens in the case of the BF models, the external sources (μ, L) are interchanged, *i.e.* the source μ associated to the nonlinear transformation of B belongs to the gauge ladder $\tilde{C}$ and vice versa.

Let us consider now the problem of identifying the anomalies which affect the Slavnov-Taylor identity (7.5) at the quantum level. We look then at the solution of the descent equations

$$\begin{aligned} b\,\omega^1_2 + d\,\omega^2_1 &= 0 \ , \\ b\,\omega^2_1 + d\,\omega^3_0 &= 0 \ , \\ b\,\omega^3_0 &= 0 \ . \end{aligned} \tag{7.19}$$

As it has been proven in refs. [10, 28], the cohomology of the BRS operator in the sector of the zero-forms with ghost number three contains, in the present case, a unique element given by

$$\omega^3_0 = C\,\partial\,C\,\partial^2\,C \ . \tag{7.20}$$

From the zero curvature condition (7.14), it follows then that the generalized cocycle of total degree three

$$\tilde{\omega}^3 = \tilde{C}\,\partial\,\tilde{C}\,\partial^2\,\tilde{C} \ , \tag{7.21}$$

belongs to the cohomology of $\widetilde{d}$. The expansion of $\tilde{\omega}^3$ will give thus a solution of the ladder (7.19), *i.e.*

$$\tilde{\omega}^3 = \omega^3_0 + \omega^2_1 + \omega^1_2 \ , \tag{7.22}$$

with ω^2_1, ω^1_2 given respectively by

$$\begin{aligned} \omega^2_1 &= (C\,\partial\,C\partial^2\,\mu - C\,\partial^2\,C\,\partial\mu + \mu\,\partial\,C\partial^2\,C)\,d\bar{z} + (\partial\,C)\,(\partial^2\,C)\,dz \ , \\ \omega^1_2 &= (-\partial\,C\partial^2\,\mu + \partial\,\mu\partial^2\,C)\,dz\wedge d\bar{z} \ . \end{aligned} \tag{7.23}$$

In particular,

$$\int \omega^1_2 = 2\int dz d\bar{z}\,C\partial^3\mu \tag{7.24}$$

is recognized to be the well known two-dimensional diffeomorphism anomaly characterizing the central charge of the energy-momentum current algebra.

Let us conclude by remarking that the complete B-C action (7.4) can be written, in perfect analogy with eq. (6.12), as

$$S = \int \widetilde{\mathcal{B}}_{z\,z} \left(d\tilde{C}^z - \tilde{C}^z \partial \tilde{C}^z \right) dz \Big|_2^0 = - \int \widetilde{\mathcal{B}}_{z\,z}\, b\, \tilde{C}^z dz \Big|_2^0 \ , \tag{7.25}$$

showing that the B-C model can be interpreted as a kind of two-dimensional BF system.

Conclusion

The zero curvature formulation of models characterized by means of a complete ladder field can be obtained as a consequence of the existence of the operator δ realizing the decomposition (1.5). Moreover, the zero curvature condition enables us to encode into a unique equation all the relevant informations concerning the BRS cohomology classes.

References

[1] O. Piguet and S.P. Sorella, *Algebraic Renormalization*, Monographs Series, m 28, Springer-Verlag, Berlin, 1995;

[2] S.P. Sorella, *Comm. Math. Phys.* 157 (1993) 231;

[3] S.P. Sorella and L.Tataru, *Phys. Lett.* B324 (1994) 351;

[4] M. Werneck de Oliveira and S.P. Sorella, *Int. J. Mod. Phys.* A9 (1994) 2979;
O. Moritsch, M. Schweda and S.P. Sorella, *Class. Quantum Grav.* 11 (1994) 1225;
O. Moritsch and M. Schweda, *Helv. Phys. Acta* 67 (1994) 289;
P.A. Blaga, O. Moritsch, M. Schweda, T. Sommer, L. Tataru and H. Zerrouki, *Algebraic structure of gravity in Ashtekar variables*, REP.TUW 94-19;
O. Moritsch, M. Schweda, T. Sommer, L. Tataru and H. Zerrouki, *BRST cohomology of Yang-Mills gauge fields in the presence of gravity in Ashtekar variables*, REP.TUW 94-17;

[5] G. Bandelloni, *Nuovo Cim.* A88 (1985) 1; *Nuovo Cim.* A88 (1985) 31;

[6] F. Brandt, N. Dragon and M. Kreuzer, *Nucl. Phys.* B340 (1990) 187;
F. Brandt, *Structure of BRS Invariant Local Functionals*, NIKHEF-H-93-12, hep-th/9310123;
G. Barnich, F. Brandt and M. Henneaux, *Local BRST cohomology in Einstein-Yang-Mills theory*, KUL-TF-95/16, ULB-TH-95/07, hep-th/9505173;

[7] E. Guadagnini, N. Maggiore and S.P. Sorella, *Phys. Lett.* B255 (1991) 65;
C. Lucchesi, O. Piguet and S. P. Sorella, *Nucl. Phys.* B395 (1993) 325;
D. Birmingham and M. Rakowski, *Phys. Lett.* B275 (1992) 289;
D. Birmingham and M. Rakowski, *Phys. Lett.* B289 (1992) 271;

[8] C. Lucchesi, O. Piguet and S.P. Sorella, *Nucl. Phys.* 395 (1993) 325;
O. Piguet, *On the Role of Vector Supersymmetry in Topological Field Theory*, UGVA-DPT 1995/02-880, hep-th/9502033;

[9] A Brandhuber, O. Moritsch, M.W. de Oliveira, O. Piguet and M. Schweda, *Nucl. Phys.* B341 (1994) 173;

[10] M. Werneck de Oliveira, M. Schweda and S.P. Sorella, *Phys. Lett.* B315 (1993) 93;
G. Bandelloni and S. Lazzarini, *J. Math. Phys.* 34 (1993) 5413;

[11] A. Boresch, M. Schweda and S.P. Sorella, *Phys. Lett.* B328 (1994) 36;

[12] M. Carvalho, L.C. Queiroz Vilar and S.P. Sorella, *Algebraic Characterization of Anomalies in Chiral W_3-Gravity*, CBPF-NF-042/94, to appear in Int. Journ. of Mod. Phys. A;

[13] J.A. Dixon, *Comm. Math. Phys.* 139 (1991) 495;

[14] J. Thierry-Mieg, *J. Math. Phys.* 21 (1980) 2834; *Nuovo Cim.* 56A (1980) 396;
L. Baulieu, *Nucl. Phys.* B241 (1984) 557;
L. Baulieu and J. Thierry-Mieg, *Phys. Lett.* B145 (1984) 53;
J. Manes, R. Stora and B. Zumino, *Comm. Math. Phys.* 102 (1985) 157;
R. Stora, *Algebraic Structure and Topological Origin of Theory* ", ed. 't Hofft et al. Plenum Press, New York, 1984;
R.D. Ball, *Phys. Rev.* 182 (1989) 1;

[15] M. Dubois-Violette, M. Talon and C.M. Viallet, *Comm. Math. Phys.* 102 (1985) 105;

[16] D. Garajeu, R. Grimm and S. Lazzarini, *W-Gauge Structure And Their Anomalies, An Algebraic Approach*, CPT-94/P.3078;

[17] D. Birmingham, M. Blau, M. Rakowski and G. Thompson, *Phys. Rev.* 209 (1991) 129;

[18] M. Green, J. Schwarz and E. Witten, *Superstring theory*, vol. I, II, *Cambridge University Press*, 1987;

[19] M. Abud, J.P. Ader and L. Cappiello, *Nuovo Cim.* 105A (1992) 1507;

[20] L. Faddeev, *Integrable Models in (1+1)-dimensional Quantum Field Theory*, Les Houches 1982, J.B. Zuber and R.Stora eds.;
A. Das, *Integrable Models*, World Scientific, Singapore, 1989;

[21] A. Das and S. Roy, *J. Math. Phys.* 31 (1990) 2145;
A. Das, W.J. Huang and S. Roy, *Int. J. Mod. Phys.* A7 (1992) 3447;
G. Akerman and R. Grimm, *J. Math. Phys.* 34 (1993) 818;
F. Brandt, *J. Math. Phys.* 35 (1994) 2463;

[22] F. Brandt, N. Dragon and M. Kreuzer, *Phys. Lett.* B231 (1989) 263; *Nucl. Phys.* B332 (1990) 224, 250;

[23] M. Dubois-Violette, M. Henneaux, M. Talon and C.M. Viallet, *Phys. Lett.* B289 (1992) 361;

[24] T. Eguchi, B. Gilkey and J. Hanson, *Phys. Rev.* 6 (1980) 213;

[25] G. Barnich, F. Brandt and M. Henneaux, *Local BRST cohomology in the antifield formalism: I. General theorems*, ULB-TH-94/06, NIKHEF-H 94-13, hepth/9405109;
G. Barnich, F. Brandt and M. Henneaux, *Local BRST cohomology in the antifield formalism: II theory*, ULB-TH-94/07, NIKHEF-H 94-15;

[26] I. A. Batalin and G. A. Vilkovisky, *Phys. Lett.* B102 (1981) 27;
I. A. Batalin and G. A. Vilkovisky, *Phys. Rev.* D28 (1983) 2567;

[27] G.T. Horowitz, *Comm. Math. Phys.* 125 (1989) 417;
G.T. Horowitz and M. Srednicki, *Comm. Math. Phys.* 130 (1990) 83;
M. Blau and G. Thompson, *Ann. Phys. (N.Y.)* 205 (1991) 130;

[28] F. Brandt, W. Troost and A. Van Proeyen, NIKHEF-H 94-16, KUL-TF 94/17, hep-th/9407061, proceedings of the workshop *Geometry of Constrained Dynamical Systems*, Isaac Newton Institute for Mathematical Sciences, Cambridge, june 1994;